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The hiring subsidy *cum* firing tax in a search model of unemployment

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Abstract

We study the macroeconomic and welfare effects of a tax-subsidy scheme on labour in a model with search unemployment. In a second-best world welfare increases if unemployment is inefficiently high or if there are pre-existing fiscal distortions. **JEL classification codes:** J3, J680. **Keywords:** hiring subsidy, firing tax, job search, equilibrium unemployment.

1 Introduction

We analyze the macroeconomic and welfare effects of a hiring subsidy *cum* firing tax scheme. Under this scheme job matches are subsidized and separations are taxed at the same amount. Pissarides' (1990) search-theoretic model of the labour market is extended and embedded within a simple macroeconomic model of a small open economy with endogenous labour supply. Although a substantial literature exists on hiring subsidies, on the one hand, and firing costs on the other hand,¹ the literature has not addressed both in a fully specified macroeconomic model.

The model shows that a tax-subsidy on labour pushes up economy-wide wages, increases vacancies, and reduces the equilibrium unemployment rate. Intuitively, the firm receiving

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¹See Mortensen and Pissarides (1999a,b) and Snower and de la Dehesa (1998) for an overview of this literature. Millard and Mortensen (1997) and Mortensen and Pissarides (1998)—employing an extension of Pissarides' model that features an endogenous job destruction process—have shown that hiring subsidies reduce the duration of unemployment but have an ambiguous effect on the unemployment rate.

the subsidy when it hires a worker can deposit it at a bank at the market rate of interest so when the time comes to pay the firing tax it is left with the accrued interest as a net subsidy. Welfare increases in a second-best world if the initial unemployment rate is inefficiently high or if there are pre-existing fiscal distortions.

The paper proceeds as follows. Section 2 sets out the model. Section 3 derives the macroeconomic effects of a marginal change in the tax-subsidy rate. Section 4 discusses the welfare effects, and section 5 concludes.

2 The model

2.1 Households

The representative household consists of infinitely many members and each member cares only about the lifetime utility achieved by the household. Individual household members experience risk in the labour market which is fully insured within the household. Since there are infinitely many members, household income is non-stochastic.² Lifetime utility of the representative household is denoted by $\Lambda(t)$:

$$\Lambda(t) \equiv \int_t^\infty \log X(\tau) e^{\rho(t-\tau)} d\tau, \quad \rho > 0, \quad (1)$$

where ρ is the pure rate of time preference, and X is *full consumption* which depends positively on goods consumption (C) and negatively on labour force participation ($U + L$):³

$$X \equiv C - \frac{[U + L]^{1+1/\sigma}}{1 + 1/\sigma}, \quad \sigma \geq 0, \quad (2)$$

where L and U stand for units of time spent on, respectively, working and on searching for a job.⁴ The household has a time endowment of unity so that leisure, M , is equal to $M \equiv 1 - U - L$. At each instant of time some unemployed household members find a job but some employed members lose their job as idiosyncratic shocks destroy a constant proportion of the pre-existing matches between firms and workers. As a result, the household's stock of employment evolves according to:

$$\dot{L} \equiv fU - sL, \quad (3)$$

where $\dot{L} \equiv dL/d\tau$, f is the matching rate (to be determined below) and s is the exogenous job destruction rate.⁵

²This assumption is quite standard in the macroeconomic literature. See, for example, Andolfatto (1996), Merz (1995), Galí (1996), Den Haan et al. (1997), Shi and Wen (1997, 1999).

³The form of the felicity function was suggested (albeit in a different context) by Greenwood, Huffman, and Hercowitz (1988). In essence, the functional form eliminates the wealth effect from the labour market participation function. See equation (9) below.

⁴The time index is dropped where no confusion arises.

⁵We assume an exogenous job destruction rate to keep the model as simple as possible. Mortensen and Pissarides (1994, 1999a) and Merz (1997) show how s can be endogenized.

The household's budget identity is:

$$\dot{A} \equiv rA + W(1 - t_L)L + s_U U - T - C, \quad (4)$$

where $\dot{A} \equiv dA/d\tau$, A is the stock of real tangible assets, r is the given real world rate of interest,⁶ W is the before-tax wage rate, t_L is the labour income tax, s_U is the unemployment benefit (a subsidy on job searching), and T is a lump-sum tax.

The household chooses time paths for consumption, searching time and tangible assets in order to maximize lifetime utility (1) subject to the accumulation identities (3)-(4) and various transversality conditions. It takes as given its initial stocks of financial assets and employment ($A(t)$ and $L(t)$). The key equations characterizing optimal household behaviour are:⁷

$$\frac{\dot{X}}{X} = r - \rho, \quad (5)$$

$$\lambda_L(t) \equiv \int_t^\infty [W(\tau)(1 - t_L) - s_U] \exp \left[- \int_t^\tau [s + f(\mu) + r] d\mu \right] d\tau, \quad (6)$$

$$= \int_t^\infty [W(\tau)(1 - t_L) - W_R(\tau)] e^{(s+r)(t-\tau)} d\tau, \quad (7)$$

$$X(t) = \rho [A(t) + H(t)], \quad (8)$$

$$U + L = W_R^\sigma, \quad W_R \equiv s_U + f\lambda_L, \quad (9)$$

where $\lambda_L(t)$ is the pecuniary value of an additional job in the planning period, W_R is the reservation wage and $H(t)$ is after-tax human wealth:

$$H(t) \equiv \int_t^\infty \left[W(\tau)(1 - t_L)L(\tau) + s_U U(\tau) - \frac{\sigma W_R(\tau)^{1+\sigma}}{1 + \sigma} - T(\tau) \right] e^{r(t-\tau)} d\tau, \quad (10)$$

$$= \lambda_L(t)L(t) + \lambda_U(t) - \int_t^\infty T(\tau) e^{r(t-\tau)} d\tau, \quad (11)$$

$$\lambda_U(t) \equiv \left(\frac{1}{1 + \sigma} \right) \int_t^\infty W_R(\tau)^{1+\sigma} e^{r(t-\tau)} d\tau. \quad (12)$$

Equation (5) is the usual Euler equation relating the optimal time profile for full consumption to the difference between the interest rate and the pure rate of time preference. As is well known in the macroeconomic literature, the representative-agent model for the small open economy only has a meaningful steady-state solution if the interest rate equals the rate of pure time preference, i.e. if $r = \rho$ (Turnovsky, 1997). It follows from (5) that there is no transition in full consumption in that case ($\dot{X} = 0$). Expression (6) shows that the pecuniary value of an additional job at time t equals the present value of the “dividend” earned on the job (equalling the excess of the after-tax wage over the unemployment benefit) using

⁶The domestic economy is small in world capital markets, so that the world real rate of interest r is exogenously given.

⁷Details of all computations are given in Heijdra and Ligthart (2001) which can be downloaded from the first author's web page.

$s+f+r$ as the instantaneous discount rate. (An equivalent expression involving the reservation wage is given in (7)). Equation (8) is the closed-form expression for full consumption in the planning period showing that the household consumes a constant proportion of its total wealth. Equation (9) shows that labour supply depends positively on the reservation wage, W_R . By participating in the labour market, rather than enjoying leisure, the household not only receives the unemployment benefit, s_U , but also has a non-zero probability, f , of locating a job with a pecuniary value of λ_L . Equation (10) shows that human wealth is the present value of the “surplus” derived from labour market participation minus lump-sum taxes. This surplus consists of the after-tax revenues from employment and search activities minus the felicity cost of supplying the optimal amount of hours to the labour market.⁸ Finally, (11)-(12) show the relationship between human wealth, the value of a job, and the initial employment level.

2.2 Firms

There is a large (and fixed) number of identical firms in the economy. To save on notation we normalize the number of such firms to unity. Following Pissarides (1990, p. 22) we assume that each firm is large so that it faces certain flows into and out of its labour force. The representative firm is perfectly competitive and uses labour (L) to produce units of the homogeneous good (Y):⁹

$$Y = \omega_0 L, \quad (13)$$

where ω_0 is the given level of labour productivity. The firm faces linear costs of adjusting its stock of labour. In order to augment its work force it must post vacancies (V) in order to find a worker. The firm’s labour force thus changes according to:

$$\dot{L} = qV - sL, \quad (14)$$

where q is the instantaneous probability of the firm finding a worker with whom it concludes a deal ($1/q$ is thus the expected duration of a vacancy). In addition to finding new workers at each instant, the firm also loses a given proportion of its work force due to idiosyncratic shocks (see also (3) above).

The objective function of the firm is the present value of its cash flow:

$$A_P(t) = \int_t^\infty \left[Y(\tau) - \gamma_V V(\tau) - W(\tau)L(\tau) + s_V \dot{L}(\tau) \right] e^{r(t-\tau)} d\tau, \quad (15)$$

where γ_V is the flow cost per vacancy (modelled in terms of lost output), and s_V is a subsidy on net hirings. If \dot{L} is negative the firm must pay the government, so s_V is also a firing tax.

⁸If labour supply is exogenous ($\sigma = 0$) there is no disutility from labour market participation and the term involving W_R drops out of (10).

⁹We abstract from capital to keep the model as simple as possible.

The firm chooses time paths for output, vacancies and employment in order to maximize (15) subject to the production function (13) and the accumulation identity for workers (14), taking as given its initial labour force ($L(t)$). Optimal firm behaviour is characterized by:

$$\mu_L + s_V = \frac{\gamma_V}{q}, \quad (16)$$

$$A_P(t) = \mu_L(t)L(t), \quad (17)$$

$$\mu_L(t) = \int_t^\infty [\omega_0 - W(\tau) - ss_V] e^{(s+r)(t-\tau)} d\tau, \quad (18)$$

where $\mu_L(t)$ is the value to the firm of an additional worker in the planning period. According to (16) the firm sets its vacancies such that the expected costs of recruitment per worker (right-hand side) equals the value to the firm of that worker *inclusive of the hiring subsidy* (left-hand side). Equation (17) shows that the stock market value of the firm is equal to the replacement value of its labour force. Finally, (18) shows that the value of an occupied job to the firm is equal to the present value of the “dividend” it earns on that job, using $s + r$ as the instantaneous discount rate. This dividend consists of the excess of labour productivity over the wage (that is, $\omega_0 - W$) minus the expected firing tax (ss_V).

2.3 Job matching and wage bargaining

Firms with vacancies and job-seeking workers are matched in a random fashion. If there are U job seekers and V vacancies then the number of contacts, X , at each instant of time is given by the matching function $X \equiv U^\epsilon V^{1-\epsilon}$, with $0 < \epsilon < 1$. Defining *labour market tightness* as $\theta \equiv V/U$ we find that the job-finding rate for the worker and the worker-finding rate for the firm are given by:

$$f(\theta) \equiv \frac{X}{U} = \theta^{1-\epsilon}, \quad q(\theta) \equiv \frac{X}{V} = \theta^{-\epsilon}, \quad (19)$$

from which it follows that $f(\theta) = \theta q(\theta)$, $f' > 0 > f''$, and $q' < 0 < q''$. Defining ϵ as the (absolute value of the) elasticity of the $q(\theta)$ function ($0 < \epsilon \equiv -\theta q'(\theta)/q(\theta) < 1$) we find that $1 - \epsilon_M$ is the elasticity of the $f(\theta)$ function.

When a firm with a vacancy and a job-seeking worker meet, a pure economic rent is created equal to $\lambda_L^i + \mu_L^i + s_V$, where the superscript i refers to a particular worker-firm pairing. Following standard practice in this literature (see, e.g., Pissarides (1990, p. 11)), we assume that this rent is shared across the two parties according to the generalized Nash wage-bargaining solution. The wage in the planning period is thus:

$$W^i(t) = \arg \max \left[[\lambda_L^i(t)]^\zeta [\mu_L^i(t) + s_V]^{1-\zeta} \right], \quad 0 < \zeta < 1, \quad (20)$$

where ζ and $1 - \zeta$ are the bargaining weights of, respectively, the worker and the firm, and where $\lambda_L^i(t)$ and $\mu_L^i(t)$ are obtained from, respectively, (7) and (18) by substituting $W = W^i$.

The wage resulting from this bargaining process can be written in two equivalent ways:

$$W = \zeta[\omega_0 + rs_V] + (1 - \zeta) \left(\frac{W_R}{1 - t_L} \right) \quad (21)$$

$$= \zeta[\omega_0 + rs_V + \gamma_V\theta] + (1 - \zeta) \left(\frac{s_U}{1 - t_L} \right). \quad (22)$$

Since all worker-firm pairings are identical and wages are renegotiated at each instant,¹⁰ the model is symmetric and the wage does not feature a pairing index i . According to (21), the wage equals the weighted average of, on the one hand, the marginal product of labour plus the *interest income* from the hiring subsidy ($\omega_0 + rs_V$) and, on the other hand, the tax-adjusted reservation wage ($W_R/(1 - t_L)$). Equation (22) shows that the wage can also be expressed as a weighted average of the tax-adjusted unemployment benefit ($s_U/(1 - t_L)$) and the firm's "surplus" ($\omega_0 + rs_V + \gamma_V\theta$). The latter consists of not only the marginal product of labour plus the interest earnings on the hiring subsidy but also includes the search costs that are foregone if the deal is struck ($\gamma_V\theta$).¹¹

2.4 Model closure

In the absence of public debt, the government budget constraint is given by:

$$t_L WL + T = s_U U + s_V \dot{L}. \quad (23)$$

The outlays on unemployment benefits and the vacancy subsidy (*cum* firing tax) are covered by the revenue from the pre-existing labour income tax and by lump-sum taxes. An immediate implication of (23) is that in the steady-state, with $\dot{L} = 0$, the hiring subsidy *cum* firing tax does not represent a net resource requirement for the government.

Asset market equilibrium ensures that household wealth equals the value of outstanding financial assets:

$$A \equiv A_P + A_F, \quad (24)$$

where A_P are shares in the domestic firms and A_F are net foreign assets. By differentiating (24) with respect to time and noting (4), (15), and (23), we obtain the expression for the current account: $\dot{A}_F = rA_F + [Y - \gamma_V V - C]$, where the term in square brackets is the trade account showing that domestic output less domestic absorption equals net exports. Domestic absorption consists of consumption by households plus investment in the stock of employment.

¹⁰In renegotiations the match surplus is still given by $\mu_L^i + \lambda_L^i + s_V$ but now s_V represents firing costs that are foregone if the match is continued. See Mortensen and Pissarides (1998, p. 8) on this point.

¹¹Equation (22) is obtained from (21) by noting that in the symmetric equilibrium W_R is equal to:

$$W_R = s_U + \left(\frac{\zeta(1 - t_L)}{1 - \zeta} \right) \gamma_V \theta.$$

Note also that $\gamma_V \theta \equiv \gamma_V V/U$ so that $\gamma_V \theta$ is the average hiring cost per job seeker.

Making use of the transversality condition of the economy as a whole, $\lim_{\tau \rightarrow \infty} e^{r(t-\tau)} A_F(\tau) = 0$, we get:

$$A_F(t) = - \int_t^\infty \left[Y(\tau) - \gamma_V V(\tau) - C(\tau) \right] e^{r(t-\tau)} d\tau. \quad (25)$$

To the extent that the country is a net creditor to the rest of the world ($A_F(t) > 0$) it can afford to run current account deficits in the future. National solvency is retained provided the present value of current account deficits (the right-hand side of (25)) equals the initial level of net foreign assets (the left-hand side of (25)).

3 The vacancy subsidy *cum* firing tax

In this section we study the macroeconomic effects of an increase in the vacancy subsidy *cum* firing tax. As was pointed out above, we assume that the interest rate equals the rate of pure time preference ($r = \rho$) so that there is no transition in full consumption ($\dot{X} = 0$). From the labour market part of the model we can prove the following proposition.

Proposition 1 *Without anticipation effects, there is no transitional dynamics in θ , q , f , μ_L , λ_L , W_R , and W . These variables jump to their new steady-state values following any unanticipated and permanent shock to any of the parameters (s , r , ω_0 , γ_V , ϵ) or policy variables (s_U , s_V , t_L).*

Proof. By using (16), (18)-(19), and (22) we obtain the following nonlinear differential equation in θ :

$$\epsilon \left(\frac{\dot{\theta}}{\theta} \right) = s + r + \left[\left(\frac{1-\zeta}{\gamma_V} \right) \left(\frac{s_U}{1-t_L} - \omega_0 - r s_V \right) + \zeta \theta \right] \theta^{-\epsilon}.$$

It follows from this expression that $\partial \dot{\theta} / \partial \theta > 0$ so that with time-invariant shocks the only economically sensible solution is for θ to equal its steady-state value. \square

The determination of the steady-state can be illustrated with the aid of Figure 1. The wage setting curve, WS, is defined in (22) and is upward sloping. By combining (16) and (18)-(19) and noting Proposition 1 (viz. that $\dot{\mu}_L = 0$), the vacancy creation curve, VC, can be written as follows:

$$\frac{\omega_0 + r s_V - W}{s + r} = \gamma_V \theta^\epsilon. \quad (26)$$

The VC curve is downward sloping and convex towards the origin (as $0 < \epsilon < 1$). The initial steady state is in point E_0 . An increase in the hiring subsidy shifts up the WS curve (from WS_0 to WS_1) as workers are able to bargain for a higher wage. At the same time the policy measure stimulates the creation of vacancies which leads to an upward shift in the VC curve

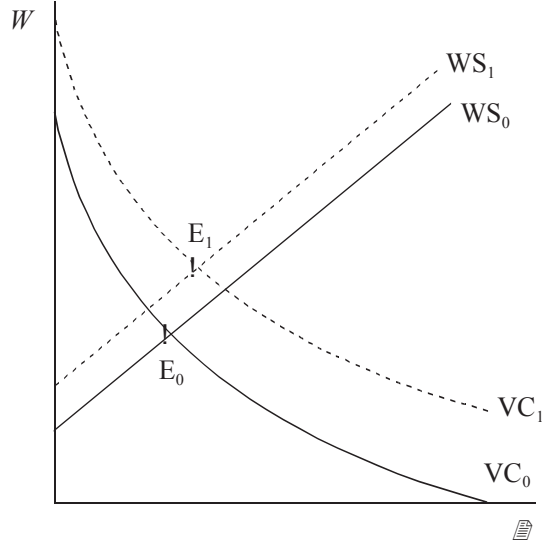


Figure 1: The hiring subsidy *cum* firing tax

(from VC_0 to VC_1) which dominates the shift in the WS curve. The new steady state is at E_1 which lies north-east from E_0 .

Denoting steady-state values with an asterisk superscript, we find—by using (22) and (26)—that:

$$\frac{d\theta^*}{ds_V} = \frac{(1 - \zeta)rf^*}{\gamma_V [\zeta f^* + \epsilon(s + r)]} > 0, \quad (27)$$

$$\frac{dW^*}{ds_V} = \zeta r \left(\frac{f^* + \epsilon(s + r)}{\zeta f^* + \epsilon(s + r)} \right) > 0, \quad (28)$$

where f^* is the steady-state job-finding rate of workers. The increase in θ^* makes it easier for the unemployed to locate a job ($df^*/ds_V > 0$) and leads to increases in the reservation wage ($dW_R^*/ds_V > 0$) and (if $\sigma > 0$) labour market participation ($d(U + L)^*/ds_V > 0$). By using (9) in the steady-state version of (3) we derive:

$$L^* = (1 - u^*) (W_R^*)^\sigma, \quad U^* = u^* (W_R^*)^\sigma, \quad (29)$$

where $u^* \equiv U^*/(U + L)^* = s/(s + f^*)$ is the steady-state unemployment *rate*. Steady-state employment rises unambiguously, both because the unemployment rate falls and (if $\sigma > 0$) because labour market participation increases. The effect on steady-state search activities is ambiguous because the unemployment rate falls but labour market participation rises. If σ is small (large) the rate effect dominates (is dominated by) the participation effect so that job search falls (rises) in the long run. Since $V^* \equiv \theta^* U^* = \theta^* u^* (W_R^*)^\sigma$ steady-state vacancies increase unambiguously.

The policy shock induces non-trivial transitional dynamics in labour market flows. Indeed, by using (1) and (9) we find that $\dot{L} = f^* (W_R^*)^\sigma - (s + f^*)L$, or:

$$L(\tau) = \left[1 - e^{(s+f^*)(t-\tau)}\right] L^* + e^{(s+f^*)(t-\tau)} L(t), \quad (30)$$

where L^* and $L(t)$ are, respectively, steady-state and initial employment. The transition speed in the labour market is equal to $s + f^*$.¹² Assuming that the system is initially in steady-state equilibrium (so that $L(t) = L^*$) we find that employment gradually increases as a result of the policy initiative:

$$\frac{dL(\tau)}{ds_V} = \left[1 - e^{(s+f^*)(t-\tau)}\right] \left(\frac{dL^*}{ds_V}\right). \quad (31)$$

The transitional dynamics in the unemployment rate and job-seeking activities can be deduced by noting that $u = 1 - (W_R^*)^{-\sigma} L$ and $U = (W_R^*)^\sigma - L$. If labour supply is exogenous ($\sigma = 0$), the paths for the unemployment rate and job search mirror the path of employment: at impact nothing happens and over time both u and U gradually decrease. If labour supply is endogenous ($\sigma > 0$), both u and U increase at impact and gradually decline over time.

4 Welfare effects

In the previous section we have shown that an increase in the vacancy subsidy *cum* firing tax is quite successful at stimulating employment and reducing the unemployment rate in the economy. In this section we study the welfare effects of such a policy initiative. We first state the following proposition.

Proposition 2 *The market solution is efficient if the following conditions hold:*

$$\zeta = \epsilon, \quad t_L = s_V = s_U = 0.$$

Proof. In the first-best optimum the social planner chooses paths for C , U , V , and L in order to maximize (1) subject to (3), (19), and (25), taking as given the initial stocks, $L(t)$ and $A_F(t)$. The first-order conditions exactly coincide with those for the decentralized market solution if and only if the conditions stated in the proposition hold. See Heijdra and Ligthart (2001) for details. \square

The first of these conditions is the well-known Hosios (1990) condition. With the Cobb-Douglas matching function adopted in this paper, ϵ is a constant and the Hosios condition only holds in the knife-edge case for which the worker's relative bargaining power happens

¹²Shi and Wen (1999, p. 472) suggests that, on a quarterly basis, reasonable values are $s = 0.05$ and $f^* = 0.79$ (an expected unemployment duration of 1.27 quarters). These values imply that labour market transition is very fast.

to be equal to the elasticity of the matching function. If the Hosios condition holds then it follows from Proposition 2 that all subsidies and taxes should be equal to zero.

In practice, of course, advanced economies do have unemployment benefit systems and do use the labour income tax to raise revenue. In addition, the Hosios condition may not hold in reality. In the remainder of this paper we therefore study the welfare effects of a vacancy subsidy *cum* firing tax in this second-best setting. To keep matters simple, we restrict attention to the case of exogenous labour supply ($\sigma = 0$) (see Heijdra and Ligthart (2001) for the general case). We first state the following proposition.

Proposition 3 *With exogenous labour supply ($\sigma = 0$) a marginal change in the hiring subsidy *cum* firing tax affects welfare according to:*

$$\begin{aligned} \rho \left(\frac{d\Lambda(t)}{ds_V} \right) &= \left(\frac{\Psi}{X^*} \right) \left(\frac{qU^*}{s + \zeta f^* + \rho} \right) \left(\frac{d\theta^*}{ds_V} \right), \\ \Psi &\equiv \left[(\zeta - \epsilon)\omega_0 - (1 - \zeta) \left(\frac{s + \epsilon f^* + \rho}{s + f^* + \rho} \right) \left(\rho s_V - \left(\frac{s_U}{1 - t_L} \right) \right) \right], \end{aligned}$$

where $d\theta^*/ds_V$ is given in (27).

Proof. In the market equilibrium $\dot{X} = 0$ (because $r = \rho$ in the small open economy) so that welfare is proportional to the level of full consumption which itself depends on total wealth (see (8)). By using (10), (15), (23)-(24), and setting $\sigma = 0$, total wealth can be written as: $\Omega(t) = A_F(t) + \int_t^\infty [\omega_0 L(\tau) - \gamma_V V(\tau)] e^{\rho(t-\tau)} d\tau$. By substituting the transition path for employment (given in equation (30) above) into this expression we obtain:

$$\Omega(t) = A_F(t) + \left(\frac{1}{\rho} \right) \left[\left(\frac{\omega_0 + \gamma_V \theta^*}{s + f(\theta^*) + \rho} \right) [f(\theta^*) + \rho L(t)] - \gamma_V \theta^* \right],$$

where $A_F(t)$ and $L(t)$ are the pre-existing stocks of, respectively, net foreign assets and employment. By differentiating this expression with respect to θ^* and simplifying we obtain the expression stated in the proposition. Details are found in Heijdra and Ligthart (2001). \square

Proposition 3 has several interesting implications. First, we note that, since $d\theta^*/ds_V > 0$ (see (27)), the sign of the welfare effect is fully determined by the sign of Ψ . Second, the proposition confirms the first-best results discussed above. If the Hosios condition holds ($\zeta = \epsilon$) and both subsidies are zero ($s_V = s_U = 0$) then a marginal change in the hiring subsidy *cum* firing tax does not have first-order welfare effects. Intuitively, this is because the economy is in the efficient equilibrium in that case.¹³ Third, even if the Hosios condition holds it is advantageous to introduce a hiring subsidy *cum* firing tax if there is a pre-existing

¹³The pre-existing labour income tax does not matter because labour supply is exogenous. Heijdra and Ligthart (2001) show that if labour supply is endogenous ($\sigma > 0$), $\zeta - \epsilon = s_V = s_U = 0$, and $t_L > 0$, welfare increases with the introduction of a hiring subsidy *cum* firing tax ($d\Lambda(t)/ds_V > 0$). Intuitively, this result follows because in that case s_V partially offsets the distorting effect of the labour income tax rate on labour market participation.

unemployment subsidy: s_V partially offsets the distorting effect of s_U on labour market participation in that case. Fourth, an expression for the second-best optimal hiring subsidy *cum* firing tax is obtained by setting s_V such that Ψ equals zero:

$$\rho \hat{s}_V = \left(\frac{s_U}{1 - t_L} \right) + (\zeta - \epsilon) \left(\frac{\omega_0}{1 - \zeta} \right) \left(\frac{s + f^* + \rho}{s + \epsilon f^* + \rho} \right). \quad (32)$$

Equation (32) illustrates the various reasons why the hiring subsidy may be beneficial in a second-best world. It can correct for pre-existing fiscal distortions (first term on the right-hand side) or for the inefficiency originating from the structure of the labour market itself (second term).

5 Conclusions

Firms firing workers impose costs upon society in terms of unemployment benefits, retraining, and other labour market measures. The formal literature on active labour market policies has suggested using (marginal) employment subsidies or hiring subsidies as one way to deal with the problem of unemployment. This paper explores a new policy strategy—a subsidy-tax scheme on labour—as an alternative to budgetary costly employment subsidy schemes. It is shown—using a simple search-theoretic model of the labour market embedded in a dynamic macroeconomic model—that if a firm pays a tax when it fires a worker to be reimbursed when it (re)hires that or another worker, the economy-wide wage goes up, the natural rate of unemployment declines, and vacancies increase. In a second-best world welfare increases if the initial unemployment rate is inefficiently high or if there are pre-existing fiscal distortions.

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